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Effect of thermal dispersion on free convection in a fluid saturated porous medium

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ABSTRACT

The present article considers a numerical study of thermal dispersion effect on the non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium. Forchheimer extension is considered in the flow equations. The coefficient of thermal diffusivity has been assumed to be the sum of molecular diffusivity and the dispersion thermal diffusivity due to mechanical dispersion. The non-dimensional governing equations are solved by the finite element method (FEM) with a Newton–Raphson solver. Numerical results for the details of the stream function, velocity and temperature contours and profiles as well as heat transfer rates in terms of Nusselt number are obtained. The study shows that the increase in thermal dispersion coefficient of the porous medium results in more heat energy to disperse away in the normal direction to the wall. This induces more fluid to flow along the wall, enhancing the heat transfer coefficient particularly near the wall.

1. Introduction

Transport phenomena in porous media have challenged engineers and researchers for quite a bit of time in trying to adopt the appropriate framework within which solution may be attained. The pioneering work of Henri Darcy on this issue opened the road towards the understanding of the main essence of the subject. That is, we do not need detailed descriptions of the motion of the fluid within the pore structure, and even if we are able to get it we would, for the sake of our engineering applications, integrate the details to get some useful quantities. It has been realized by the late seventies of the last century that we may adopt the continuum hypothesis to transport phenomena in porous media through upscaling processes. To adopt the continuum approach to phenomena occurring in highly complex and even unknown structure implies that the primary variables of interest or its derivatives that may suffer of discontinuities at the interface between the fluid body and the solid grains of the porous media may be replaced by other variables that are continuous everywhere in the domain. This would lead us to substantially decrease the number of degrees of freedom of the system and hence made it amenable to mathematical manipulations. However, to correctly adopt the continuum hypothesis to transport phenomena in porous media, certain conditions and length scale constraints need to be satisfied. Failing to satisfy these conditions would result in the inappropriateness of the continuum hypothesis and other sophisticated methods need to be adopted (Salama and Van Geel, 2008).

Yet for all the simplifications that it made, the continuum hypothesis as applied to transport phenomena in porous media has confronted some difficulties that entailed the introduction of some constitutive relationships to account for the apparent differences between the upscaled and the actual variables. To give an example, the actual velocity of fluid particles within the pore structures changes significantly between zero at the interface and different than zero within the pores, whereas within the continuum hypothesis the upscaled velocity may be constant or at most changes, comparatively, slowly. This would result in the existence of additional mass and/or energy fluxes. It has thus been hypothesized that these additional fluxes may be accounted for by adding terms to their respective flux terms that may be assumed to depend on the upscaled velocity. Thus, in terms of solute transport, the usual diffusion mechanism has been augmented by another mechanism that is called mass dispersion term which depends on the upscaled velocity. Likewise, energy transport (like heat transport) in porous media required the addition of a thermal dispersion mechanism to the usual thermal diffusion mechanism.

In this study, we consider the effect of thermal dispersion mechanism on the development of the thermally-driven convection boundary layer flow in porous media. This subject is of considerable interest in a variety of engineering applications including geothermal energy technology, petroleum recovery, filtration processes, packed bed reactors and underground disposal of chemical and nuclear waste.

Similarity solution for the Darcian regime with no dispersion has been presented by Cheng and Minkowycz (1977). The

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Nomenclature

A	constant
C	empirical constant
d	pore diameter
g	gravitational constant
K	permeability of the porous medium
k_d	dispersion thermal conductivity
k_e	effective thermal conductivity
Nu_x	local Nusselt number
p	pressure
q	local heat flux
Ra	Rayleigh number
$\overline{T}$	temperature
T	non-dimensional temperature
$\overline{u}, \overline{v}$	velocity components in the $\overline{x}$ and $\overline{y}$ directions
u, v	non-dimensional velocity components in the x and y directions
$\overline{x}, \overline{y}$	Cartesian coordinates
x, y	non-dimensional Cartesian coordinates
ρ	fluid density
μ	viscosity
v	fluid kinematic viscosity
α	molecular thermal diffusivity
α_d	dispersion diffusivity
α_x, α_y	thermal diffusion coefficients in x and y directions respectively
β	thermal expansion coefficient
γ	mechanical dispersion coefficient
ψ	dimensional stream function

Subscripts

w	evaluated on the wall
∞	evaluated at the outer edge of the boundary layer

dependence of heat dispersion on the upscaled velocity has been proposed by several investigators to be linear (e.g., Fried and Combarnous (1976), Georgiadis and Catton (1988), Cheng (1981) and Plumb (1983)). On the other hand, an analysis of the effect of thermal dispersion on vertical plate natural convection in porous media is presented by Hong and Tien (1987). Lai and Kulacki (1989) investigated the effect of thermal dispersion on the non-Darcy convection from a horizontal surface submerged in saturated porous media. The effects of thermal dispersion and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium were studied by Murthy and Singh (1997). Mansour and El-Amin (1999) studied the effects of thermal dispersion on non-Darcy axisymmetric free convection in a saturated porous medium with lateral mass transfer. El-Amin (2004) investigated the effects of double dispersion on natural convection heat and mass transfer in non-Darcy porous medium. The problem of thermal dispersion effects on non-Darcy axisymmetric free convection in a power-law fluid saturated porous medium was studied by El-Amin (2005).

The present investigation is devoted to study the effect of thermal dispersion on Forchheimer natural convection over a vertical flat plate in a fluid saturated porous medium. The coefficient of thermal diffusivity is assumed to be the sum of molecular diffusivity and the dispersion thermal diffusivity due to mechanical dispersion. The wall temperature distribution is assumed to be uniform. The non-dimensional equations are solved using the FEM.

2. Analysis

Let us consider the non-Darcy natural convection flow and heat transfer over a semi infinite vertical surface in a fluid saturated porous medium, Fig. 1. The governing equations for this problem are given by

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{1}$$

$$\overline{u} + \frac{C\sqrt{K}}{v}\overline{u}q = -\frac{K}{\mu}\left(\frac{\partial p}{\partial \overline{x}} + \rho g\right) \tag{2}$$

$$\overline{v} + \frac{C\sqrt{K}}{v}\overline{v}q = -\frac{K}{\mu}\left(\frac{\partial p}{\partial \overline{y}}\right) \tag{3}$$

$$\overline{u}\frac{\partial \overline{T}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{T}}{\partial \overline{y}} = \frac{\partial}{\partial \overline{x}}\left(\alpha_{\overline{x}}\frac{\partial \overline{T}}{\partial \overline{x}}\right) + \frac{\partial}{\partial \overline{y}}\left(\alpha_{\overline{y}}\frac{\partial \overline{T}}{\partial \overline{y}}\right) \tag{4}$$

$$\rho = \rho_\infty[1 - \beta(\overline{T} - \overline{T}_\infty)] \tag{5}$$

where $q^2 = \overline{u}^2 + \overline{v}^2$ along with the boundary conditions

$$\begin{aligned} &\overline{y} = 0 : \overline{v} = 0, \quad \overline{T}_w = const. \\ &\overline{y} \to \infty : \overline{u} = 0, \quad \overline{T} \to \overline{T}_\infty \end{aligned} \tag{6}$$

where $\overline{u}$ [L/T] and $\overline{v}$ [L/T] are the velocity components in the $\overline{x}$ and $\overline{y}$ -directions, respectively, $(\rho_\infty C_p)_f$ is the product of density [M/L^3] and specific heat of the fluid [L^2/KT^2], p is the pressure [M/LT^2], $\overline{T}$ is the temperature [K], K [L^2] is the permeability constant, C is an empirical constant, dimensionless, β is the thermal expansion coefficient [1/K], μ is the viscosity of the fluid [M/LT], g is the acceleration due to gravity [L/T^2], $\alpha_{\overline{x}}$ and $\alpha_{\overline{y}}$ are the components of the thermal diffusivity in $\overline{x}$ and $\overline{y}$ directions respectively [L^2/T]. The normal component of the velocity near the boundary is considered small compared with the vertical component and the derivatives of any quantity in the normal direction to the wall are large compared with derivatives of the quantity in the direction of the wall (i.e., the vertical direction). Under these assumptions Eqs. (1)–(5) become:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{7}$$

$$\overline{u} + \frac{C\sqrt{K}}{v}\overline{u}|\overline{u}| = -\frac{K}{\mu}\left(\frac{\partial p}{\partial \overline{x}} + \rho g\right) \tag{8}$$

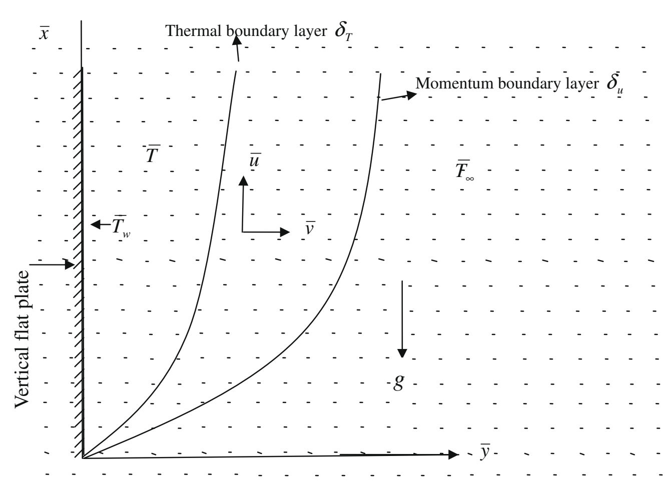

Fig. 1. Physical model and coordinate system.

$$\frac{\partial p}{\partial \bar{y}} = 0 \quad (9)$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(\alpha_{\bar{y}}\frac{\partial \bar{T}}{\partial \bar{y}}\right) \quad (10)$$

The quantity $\alpha_{\bar{y}}$ is variable and is defined as the sum of molecular thermal diffusivity α and dispersion thermal diffusivity α_d. Following Plumb (1983), we assume the dispersion thermal diffusivity as to linearly change with the upscaled velocity such that, $\alpha_d = \gamma|\bar{u}|d$, where γ is the mechanical dispersion coefficient whose value depends on the structure of the porous media and d is the pore diameter.

Table 1
Grid independent test (R_{ad} = 10, γ = 0.5, F_0 = 0.5, x = 10, y = 10).

Mesh size	ψ	T
10 × 10	1.181790E+001	1.507303E−001
20 × 20	1.234669E+001	2.080401E−001
30 × 30	1.251212E+001	2.240477E−001
40 × 40	1.259287E+001	2.318501E−001
50 × 50	1.264069E+001	2.364608E−001
60 × 60	1.267229E+001	2.395064E−001
70 × 70	1.269473E+001	2.416684E−001

Invoking the Boussinesq approximations, by substituting Eq. (5) into Eq. (9), and with some mathematical manipulations, the pressure term can be eliminated and the velocity components u and v can be written in terms of stream function $\bar{\psi}$ as: $\bar{u} = \partial\bar{\psi}/\partial\bar{y}$ and $\bar{v} = -\partial\bar{\psi}/\partial\bar{x}$, we obtain:

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} + \frac{C\sqrt{K}}{\nu}\frac{\partial}{\partial \bar{y}}\left(\frac{\partial \bar{\psi}}{\partial \bar{y}}\right)^2 = \frac{Kg\beta}{\nu}\frac{\partial \bar{T}}{\partial \bar{y}} \quad (11)$$

$$\frac{\partial \bar{\psi}}{\partial \bar{y}}\frac{\partial \bar{T}}{\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left[(\alpha + \alpha_d)\frac{\partial \bar{T}}{\partial \bar{y}}\right] \quad (12)$$

Introducing the non-dimensional transformations:

$$y = \bar{y}/d, x = \bar{x}/d, \psi = \bar{\psi}/\alpha, T = (\bar{T} - \bar{T}_\infty)/(\bar{T}_w - \bar{T}_\infty) \quad (13)$$

The problem statement then becomes:

$$\frac{\partial^2 \psi}{\partial y^2} + 2F_0\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y^2} = Ra\frac{\partial T}{\partial y} \quad (14)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\left(1 + \gamma\frac{\partial \psi}{\partial y}\right)\frac{\partial T}{\partial y}\right] \quad (15)$$

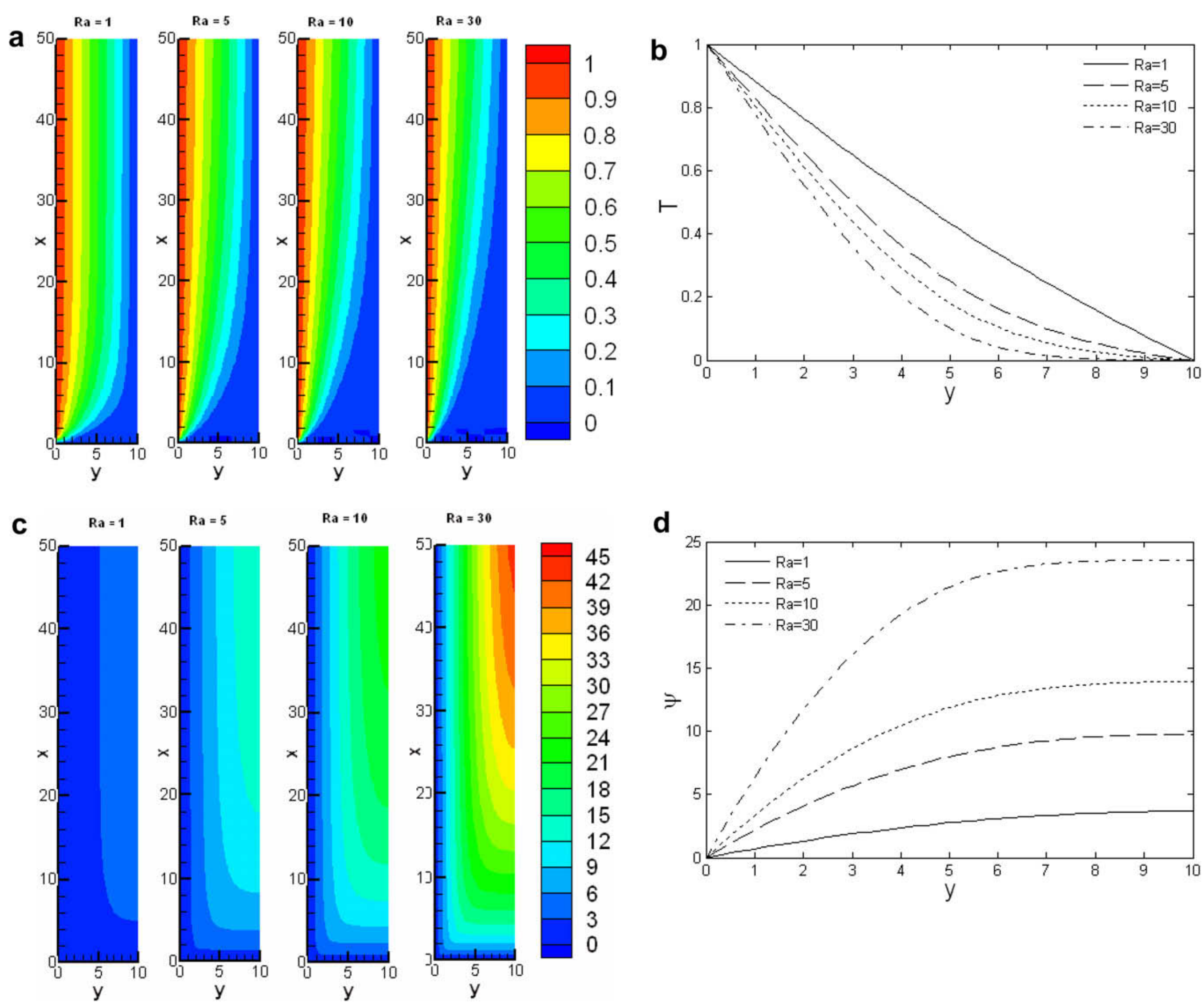

Fig. 2. (a) Temperature contours for various values of Ra at F_0 = 0.5 and γ = 0.3. (b) Temperature profiles for various values of Ra with F_0 = 0.5 and γ = 0.3 at x = 10. (c) Stream function contours as a function of y for various values of Ra at F_0 = 0.5 and γ = 0.3. (d) Stream function profiles as a function of y for various values of Ra at F_0 = 0.5 and γ = 0.3. (e) Velocity contours for various values of Ra with F_0 = 0.5 and γ = 0.3 at x = 10. (f) Velocity profiles for various values of Ra with F_0 = 0.5 and γ = 0.3 at x = 10. (g) Nusselt number as a function of x for various values of Ra with F_0 = 0.5 and γ = 0.3.